Département de génie électrique et de génie informatique
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MAT-19961 CALCUL MATRICIEL EN GÉNIE

Solutions - Devoir #10

6.3.10

$$\hat{\mathbf{y}} = \frac{(\mathbf{y} \cdot \mathbf{u}_1)}{(\mathbf{u}_1 \cdot \mathbf{u}_1)}\mathbf{u}_1 + \frac{(\mathbf{y} \cdot \mathbf{u}_2)}{(\mathbf{u}_2 \cdot \mathbf{u}_2)}\mathbf{u}_2 + \frac{(\mathbf{y} \cdot \mathbf{u}_3)}{(\mathbf{u}_3 \cdot \mathbf{u}_3)}\mathbf{u}_3$$

$$= \frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix} + \frac{14}{3}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} - \frac{5}{3}\begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 3 \\ 6 \end{bmatrix}$$

$$\mathbf{z} = \mathbf{y} - \hat{\mathbf{y}} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} - \begin{bmatrix} 5 \\ 2 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 2 \\ 0 \end{bmatrix}$$

Donc:

$$\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z} = \begin{bmatrix} 5 \\ 2 \\ 3 \\ 6 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \\ 2 \\ 0 \end{bmatrix}$$

6.3.20

Soit: $\mathbf{y} = \mathbf{u}_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

$$\hat{\mathbf{y}} = \frac{(\mathbf{y} \cdot \mathbf{u}_1)}{(\mathbf{u}_1 \cdot \mathbf{u}_1)}\mathbf{u}_1 + \frac{(\mathbf{y} \cdot \mathbf{u}_2)}{(\mathbf{u}_2 \cdot \mathbf{u}_2)}\mathbf{u}_2 = \frac{1}{6}\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} - \frac{1}{30}\begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{5} \\ -\frac{2}{5} \end{bmatrix}$$

$$\mathbf{z} = \mathbf{y} - \hat{\mathbf{y}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{1}{5} \\ -\frac{2}{5} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{4}{5} \\ \frac{2}{5} \end{bmatrix}$$

Donc un vecteur **v** peut être n'importe quel multiple non-nul de **z**.

6.3.26

$$\hat{\mathbf{b}} = UU^T\mathbf{b} = (0.20, 0.92, 0.44, 1.00, -0.20, -0.44, 0.60, -0.92)$$

La distance de **b** à Col U est alors:

$$\|\hat{\mathbf{b}} - \mathbf{b}\| = 2.1166$$

6.4.16

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & -3 & 1 \\ 0 & 2 & 3 \\ 1 & 5 & 2 \\ 1 & 5 & 8 \end{bmatrix}, \mathbf{x}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \boldsymbol{x}_2 = \begin{bmatrix} 3 \\ -3 \\ 2 \\ 5 \\ 5 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} 5 \\ 1 \\ 3 \\ 2 \\ 8 \end{bmatrix}$$

$$\mathbf{v}_1 = \mathbf{x}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{v}_2 = \mathbf{x}_2 - \frac{(\mathbf{x}_2 \cdot \mathbf{v}_1)}{(\mathbf{v}_1 \cdot \mathbf{v}_1)}\mathbf{v}_1 = \begin{bmatrix} 3 \\ -3 \\ 2 \\ 5 \\ 5 \end{bmatrix} - \frac{16}{4}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{v}_3 = \mathbf{x}_3 - \frac{(\mathbf{x}_3 \cdot \mathbf{v}_1)}{(\mathbf{v}_1 \cdot \mathbf{v}_1)}\mathbf{v}_1 - \frac{(\mathbf{x}_3 \cdot \mathbf{v}_2)}{(\mathbf{v}_2 \cdot \mathbf{v}_2)}\mathbf{v}_2 = \begin{bmatrix} 5 \\ 1 \\ 3 \\ 2 \\ 8 \end{bmatrix} - \frac{14}{4}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} - \frac{12}{8}\begin{bmatrix} -1 \\ 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 0 \\ -3 \\ 3 \end{bmatrix}$$

$$\mathbf{v}'_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

On normalise les trois vecteurs $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$' précédents, et on construit la matrice suivante dont les colonnes correspondent aux trois vecteurs normalisés:

$$Q = \begin{bmatrix} \frac{1}{2} & -\frac{1}{\sqrt{8}} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{\sqrt{8}} & \frac{1}{2} \\ 0 & \frac{2}{\sqrt{8}} & 0 \\ \frac{1}{2} & \frac{1}{\sqrt{8}} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{\sqrt{8}} & \frac{1}{2} \end{bmatrix}$$

$$R = Q^T A = \begin{bmatrix} 2 & 8 & 7 \\ 0 & 2\sqrt{2} & 6\sqrt{2} \\ 0 & 0 & 6 \end{bmatrix}$$

6.4.22

$$\mathbf{x}_1 = \begin{bmatrix} -10 \\ 2 \\ -6 \\ 16 \\ 2 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} 13 \\ 1 \\ 3 \\ -16 \\ 1 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} 7 \\ -5 \\ 13 \\ -2 \\ -5 \end{bmatrix}, \mathbf{x}_4 = \begin{bmatrix} -11 \\ 3 \\ -3 \\ 5 \\ -7 \end{bmatrix}$$

$$\mathbf{v}_1 = \mathbf{x}_1 = \begin{bmatrix} -10 \\ 2 \\ -6 \\ 16 \\ 2 \end{bmatrix}$$

$$\mathbf{v}_2 = \mathbf{x}_2 - \frac{(\mathbf{x}_2 \cdot \mathbf{v}_1)}{(\mathbf{v}_1 \cdot \mathbf{v}_1)}\mathbf{v}_1 = \begin{bmatrix} 13 \\ 1 \\ 3 \\ -16 \\ 1 \end{bmatrix} - \left(-\frac{400}{400}\right)\begin{bmatrix} -10 \\ 2 \\ -6 \\ 16 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -3 \\ 0 \\ 3 \end{bmatrix}$$

$$\mathbf{v}_3 = \mathbf{x}_3 - \frac{(\mathbf{x}_3 \cdot \mathbf{v}_1)}{(\mathbf{v}_1 \cdot \mathbf{v}_1)}\mathbf{v}_1 - \frac{(\mathbf{x}_3 \cdot \mathbf{v}_2)}{(\mathbf{v}_2 \cdot \mathbf{v}_2)}\mathbf{v}_2 = \begin{bmatrix} 7 \\ -5 \\ 13 \\ -2 \\ -5 \end{bmatrix} - \left(-\frac{200}{400}\right)\begin{bmatrix} -10 \\ 2 \\ -6 \\ 16 \\ 2 \end{bmatrix} - \left(-\frac{48}{36}\right)\begin{bmatrix} 3 \\ 3 \\ -3 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 6 \\ 6 \\ 0 \end{bmatrix}$$

$$\mathbf{v}_4 = \mathbf{x}_4 - \frac{(\mathbf{x}_4 \cdot \mathbf{v}_1)}{(\mathbf{v}_1 \cdot \mathbf{v}_1)}\mathbf{v}_1 - \frac{(\mathbf{x}_4 \cdot \mathbf{v}_2)}{(\mathbf{v}_2 \cdot \mathbf{v}_2)}\mathbf{v}_2 - \frac{(\mathbf{x}_4 \cdot \mathbf{v}_3)}{(\mathbf{v}_3 \cdot \mathbf{v}_3)}\mathbf{v}_3 = \begin{bmatrix} -11 \\ 3 \\ -3 \\ 5 \\ -7 \end{bmatrix} - \frac{200}{400}\begin{bmatrix} -10 \\ 2 \\ -6 \\ 16 \\ 2 \end{bmatrix} + \frac{36}{36}\begin{bmatrix} 3 \\ 3 \\ -3 \\ 0 \\ 3 \end{bmatrix} - \left(-\frac{54}{108}\right)\begin{bmatrix} 6 \\ 0 \\ 6 \\ 6 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 5 \\ 0 \\ 0 \\ -5 \end{bmatrix}$$

6.4.24

On peut utiliser la fonction Matlab suivante:

```
function [Q, R]=gramm(A)

% La première colonne de Q
Q=A(:,1)/norm(A(:,1));
```

```
for j=2:size(A,2)
   v=A(:,j)-Q*(Q'*A(:,j));
   Q(:,j)=v/norm(v);
   % Pour ajouter une autre colonne à Q
end

R=Q'*A;
```

Avec la matrice *A* du problème 6.4.22, cela donne:

```
>> A

A =

   -10    13     7   -11
     2     1    -5     3
    -6     3    13    -3
    16   -16    -2     5
     2     1    -5    -7

>> [Q, R]=gramm(A)

Q =

   -0.5000    0.5000    0.5774    0.0000
    0.1000    0.5000    0.0000    0.7071
   -0.3000   -0.5000    0.5774    0.0000
    0.8000         0    0.5774    0.0000
    0.1000    0.5000    0.0000   -0.7071

R =

   20.0000  -20.0000  -10.0000   10.0000
    0.0000    6.0000   -8.0000   -6.0000
   -0.0000    0.0000   10.3923   -5.1962
   -0.0000    0.0000    0.0000    7.0711
```

Exercice Matlab

1)

```
function y=monte(funfcn, n, a, b)
x=a+(b-a)*rand(1,n);
y1=feval(funfcn, x);
y=(b-a)*mean(y1);
```

2)

```
function y=fct1(x)
y = (2/sqrt(pi))*exp(-x.^2);

>> erf(1)

ans =

    0.8427

>> quad('fct1',0,1)

ans =

    0.8427

>> monte('fct1',1000,0,1)

ans =

    0.8405
```

3)

```
function y=fct2(x)
y=sin(sqrt(exp(x.^1.23)));

>> quad('fct2',0.5,1.9)

ans =

    1.1439

>> monte('fct2',1000,0.5,1.9)

ans =

    1.1391
```