PERFORMANCE ANALYSIS OF A MULTICARRIER DIGITAL AUDIO BROADCASTING SYSTEM IN A WIDE-BAND MOBILE RADIO ENVIRONMENT

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Abstract

Over the past few years, digital technologies have been growing in such a way that a new generation of audio broadcasting systems, fully digital, can now become cost effective. For this purpose Coded Orthogonal Frequency Division Multiplexing (COFDM) [1], a new transmission technique designed in Europe, is promising. In order to perform the evaluation of this system, we need an accurate channel model which takes into account the physical structure and the time varying nature of the propagation medium. The purpose of this work is to study the performances of COFDM using a wide band channel simulator based on the scattering function.

1.Introduction

The realization of a high quality digital sound broadcasting system in a time varying selective frequency channel is possible with the development of good sound data compression algorithm and with the design of a high rate digital system. This system will have to resist to deep fades throughout the band and to offer a good spectrum efficiency. This work deals with performance evaluations of such a system. In order to analyze the performances of a communication system, we need to reproduce with precision the channel characteristics. In the first part of this paper, the multipath and time varying nature of the mobile radio propagation channel are briefly described. The influence of the physical channel structure on the UHF scattering electromagnetic waves viewed by the mobile receivers is discussed. This analysis leads to a mobile radio channel model based on the scattering function of the propagation medium.

Digital technologies have been growing in such a way that more complex modulation systems are now feasible and can be cost effective. COFDM is such a technique. It combines a multicarrier modulator/demodulator, a convolutional coding system and a time-frequency interleaver. The purpose of the second part of this paper is to describe the COFDM system.

The last part of this work presents the results obtained with our simulator. Several simulations have been carried out, each of them representing specific channel conditions. This

gives a general idea of the system behavior.

2.Channel Modeling via the Scattering Function

Multipath channels are an important class of channels which includes ionospheric, acoustic and mobile radio channels. In a multipath channel, the signal viewed at the receiver is a sum of the delayed paths of the transmitted signal (eq. 1). This sum can be constructive or destructive over all the band, which can result in severe deep fades.

$$h(t, \tau) = \sum_i c_i(t) e^{-j\theta_i(t)\tau} \delta(t - \tau_i) \quad (1)$$

As the receiver is moving, these paths are combining differently over time. This fluctuation is referred to as short time fast fading. Some paths appear and disappear with time. These variations lead to slow fading. In this work, we restrain our study to short time fading.

When a receiver is moving with respect to the transmitter, the received signal is subjected to a frequency shift called Doppler shift. Assuming constant vehicle speed, the maximum Doppler shift is then given by the following relation:

$$f_d = v \cos(\theta) / \lambda \quad (2)$$

where v is the vehicle speed, λ the wavelength and θ the angle between the mobile motion and the received path. If we assume that the scatters and reflectors are uniformly distributed through the medium, this leads to the well known Jakes spectrum [2].

The wideband mobile radio channel can be characterized by the scattering function. The position of a given scatterer with respect to the vehicle, its angle from the vehicle motion, gives its position in the propagation delay and Doppler's frequency domain of the scattering function. The general topography of a propagation medium, i.e. orientation of the roads, buildings positions and the presence of hills, characterizes the profile delay of this place. Parsons and Dersh have shown [3][4] that a close relation exists between some obstacles and scatterers in the delay-Doppler scattering function (figure 1).

Figure 1 Link between scattering function and a given propagation medium

This system is designed to work at UHF frequencies. In this work, 1 GHz (corresponding to a 30 cm wavelength) was used. Each individual elementary scatterer has thus its own independent phase shift. They are then considered independent and having a uniform phase distribution leading to an uncorrelated scatterers (US) channel. On the other hand, on a mobile short run ($\sim 10\lambda$ to 40λ), the modulus of each path remains almost the same, but the complex sum of each scatterer changes, resulting in a statistical variation of the channel. Taking these facts into account leads to a wide sense stationary channel. The propagation medium is then a wide sense stationary and uncorrelated scatterer (WSSUS) channel.

This channel can be modeled by a deterministic part given by the medium topography and materials' dielectric constants and a statistical part consisting of a bidimensional (delay-Doppler) random field characterizing the phase of each elementary path. The complex random field has a gaussian distribution in phase and in quadrature with equal variance resulting in a uniform phase distribution. This channel model is given by the following equation.

$$P(\tau, f_d) = \frac{1}{2} S(\tau, f_d) |x(\tau, f_d) + jy(\tau, f_d)|^2 \quad (3)$$

where $P(\tau, f_d)$ is the channel delay-Doppler power density function, $S(\tau, f_d)$ the scattering function (deterministic part), and $x(\tau, f_d)$ and $y(\tau, f_d)$ are the complex bidimensional gaussian random field.

Mathematically, the mobile radio channel can be described by a time variant linear filter, by a time varying transfer function and also by its scattering function. These functions are related by a bidimensional Fourier transform relationship. These relations are used to obtain the channel transfer function from the channel delay-Doppler power density function (figure 2 and eq. 4).

Figure 2 Fourier transform relationship between channel power density function and $H(t, f)$

$$H(t, f) = \quad (4)$$

$$\frac{1}{\sqrt{2}} \int_{-f_{dmax}}^{f_{dmax}} \int_0^{\tau_{max}} \sqrt{S(\tau, f_d)} (x(\tau, f_d) + jy(\tau, f_d)) e^{-j2\pi f t} e^{j2\pi f_d t} d\tau df_d$$

3. COFDM Modulation Technique

Coded Orthogonal Frequency Division Multiplexing technique was recently developed in Europe. It is a multicarrier modulation (MCM) scheme [1] that splits the information over a given number of orthogonal sub-carriers at low bit rate, thus resulting in several non-selective channels [1]. Such systems are called parallel in contrast to serial ones that use only one carrier at high bit rate.

The COFDM offers a good spectrum efficiency which is of primary importance in audio broadcasting. Convolutional coding is used to minimize errors and interleaving is used to decorrelate data at the input of the Viterbi decoder. Classical COFDM systems employ a rate 1/2 convolutional code with a free distance of 10 [1]. Frequency interleaving is generated in a pseudo-random way. This technique is used to decorrelate data coming from adjacent subcarriers. A bloc interleaver is used to mix data over time. This feature is particularly useful for fast fading channels to mitigate the effect of deep fades. The use of interleaving results in a diversity gain over Rice and Rayleigh channels because it decorrelates bits at the input of the Viterbi decoder, thus avoiding burst errors.

Two systems can be realized to broadcast digital data:

coherent and differential. Coherent systems are better than differential ones, but they need an estimate of the channel transfer function. Differential systems are easier and cheaper to realize. In this work we consider only differential decoding. Each subcarrier is then carrying a QPSK-4 differentially coded symbol (DQPSK-4).

Multicarrier modulation techniques can be achieved with the use of a bank of N modulators. This operation implies serious restrictions from a hardware point of view. The modulation of N orthogonal sine waves can also be easily achieved by Fast Fourier Transforms (FFT) techniques [5]. The same scheme is used to synthesize a bank of N correlators at the receiver. The transmitted signal is then given by the following equation:

$$y(t) = \sum_{k=0}^{N-1} C_k \exp(2\pi i f_k t) \quad (5)$$

where

C_k = QPSK constellation points,
 N = number of subcarriers,
 f_k = $f_o + k / t_s$, with f_o the carrier frequency and t_s the useful symbol duration.

It can be shown that the sampled transmitted signal can be written as:

$$y(mT_e) = A \cdot \text{FFT}^{-1}(C_k) \quad (6)$$

where

A = proportionality constant,
 mT_e = sampling times at frequency $1/T_e$.

This type of modulation gives a signal composed of a sum of truncated orthogonal sine waves. The signal spectrum is then composed of a sum of *sinc* functions regularly spaced over the frequency band, resulting in a flat spectrum over that band.

In this work, European DAB project [1] COFDM parameters are used (see table 1). In this case, spectrum efficiency is reduced because a guard interval is added. Global spectrum efficiency of a DQPSK-4 system with these parameters is 0.8 bit/s/Hz. Transmitted signal bandwidth is therefore 7 MHz.

COFDM Parameters	
Data rate	5.6 Mbit/s
Guard interval	16 μ s
Useful period	64 μ s
Number of subcarriers	448
Code rate	1/2

Table 1 COFDM parameters

For the frequency and time coherence characteristics of the channel, we can fix two COFDM parameters, namely the

useful symbol duration (t_s) and the total symbol duration (T_s) which is the sum of t_s and the guard interval.

4. Simulations and Results

In our simulations, convolutional decoding was performed with the Viterbi algorithm using soft decisions. BER estimates were computed over 600 000 data bits. This number of data bits allows us to estimate a BER of 10^{-4} with a confidence interval of 95% and a relative precision of $2 \cdot 10^{-1}$.

In order to see the influence of different kinds of transmissions channels on COFDM, the form of the scattering function was varied. First, the channel spreading in the delay domain was modified. The delay domain was modeled as a decreasing exponential. The time constant of the exponential was changed. Second, a strong scatterer was introduced. Simulations were done for different positions of this scatterer. Third, the mobile receiver speed was changed, resulting in a variation of the Doppler spread.

Let us consider the effect of changing the spread of the channel in the delay domain. Simulations results are shown in figure 3. We can see that increasing the delay spread leads to a degradation of BER performances. The effect of an increase of the delay spread is to augment the channel selectivity, which results in a larger number of carriers affected by deep fades. A channel with a very small delay spread can be seen as being narrow-band. This means that all the carrier are subjected at the same time to a deep fade over all the band, causing a large number of errors. The conclusion is that COFDM systems work better in a channel that is reasonably selective in frequency.

A strong scatterer was also added to the exponential profile delay. Its delay was varied to see the effect on the overall performance. Figure 4 shows that increasing the delay of the scatterer reduces the resulting performances. The effect is even worse and seems to lead to a slow decreasing BER curves when the scatterer delay is larger than the guard interval. This can be explained by the fact that when the large scatterer is too far, it introduces intersymbol interference between adjacent symbols.

The spread in the Doppler-frequency domain was modified through the variation of the receiver's speed. Simulation results (figure 5) show that the system is not severely affected by the variation of the maximum Doppler spread. Simulation results indicate that BER performances are decreasing significantly only when the maximum Doppler spread is very large (1kHz, corresponding to a speed of 900 km/h). We can also see that the COFDM performances are better when the vehicle is moving slowly than when it is stopped. The reason for this is that the variation of fade positions introduced by the vehicle movements results in a

sort of time diversity which is good for the decoding system. When the Doppler shift is too large, the phase variations between two consecutive symbols introduce errors in the differential phase estimation process.

5. Conclusion

In this paper, we have described a wideband channel simulator and applied it to analyze COFDM performances over several mobile channel conditions. Simulations results show that COFDM works better in a channel that is reasonably selective in frequency. The effect of a strong scatterer was also studied. It was found that performances diminish as the delay of the strong scatterer is increased. Finally, it was shown that COFDM performs well over a wide range of Doppler shift values.

From these results, we can conclude that COFDM offers robust performances in a mobile environment. It is therefore a good candidate not only for DAB systems, but also for other large bandwidth systems, such as HDTV.

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Figure 3 Needed E_b/N_0 for a BER of 10^{-3} versus channel delay spread

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